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REVIEWS AND DESCRIPTIONS OF TABLES AND BOOKS

The numbers in brackets are assigned according to the American Mathematical Society classification scheme. The 1991 Mathematics Subject Classification can be found in the annual subject index of *Mathematical Reviews* starting with the December 1990 issue.

1[42C05, 41A21, 44A60, 43A17]—*Orthogonal Rational Functions*, by Adhemar Bultheel, Pablo Gonzáles-Vera, Erik Hendriksen, and Olav Njåstad, Cambridge University Press, New York, NY, 1999, xiv+407 pp., $23\frac{1}{2}$ cm, hardcover, \$59.95

The theory of orthogonal polynomials is well known and several aspects have permeated the graduate and even undergraduate curriculum of mathematics, physics and engineering.

Properties and connections with continued fractions and Padé approximation (either using a series expansion around $z = 0$, leading to polynomials in z, or around $z = \infty$ for polynomials in $1/z$; the latter intimately connected with the Stieltjes transform) are well documented in a score of textbooks and research monographs (compare a.o. Szegő [1], Chihara [2], Freud [3], Baker & Graves-Morris [4], Perron [5], Wall [6] and Jones & Thron [7]).

Generalizations leading away from the realm of nonnegative Borel measures to the field of moment functionals and formal procedures have abounded, but the most promising studies—from the viewpoint of theory and applications—can be found in the study of orthogonal rational functions with a prescribed sequence of poles in the extended complex plane (for the polynomials mentioned before, all poles are either fixed at $z = \infty$ or $z = 0$).

Two directions can be taken: starting from polynomials orthogonal on the real line or from those orthogonal on the unit circle. For the former it stands to reason to choose the finite poles on the real axis (i.e., possibly belonging to the support of the measure) and for the latter a natural location is outside the unit disk (siding with $z = \infty$).

As stated by the authors, the link between the two starting points can be given by a Cayley transform, enabling a parallel treatment to a great extent. Only the distinction between poles on or not on the support then necessitates a separate treatment.

The properties of "ordinary" orthogonal polynomials to be generalized to the rational case have, for reasons of narrowing the tremendously broad field of research, been chosen to be in the line of classical interpolation problems of Schur and Carathéodory type, quadrature formulae and moment problems.

The approach can be considered "classical": from a Christoffel–Darboux formula one derives a recurrence relation for the orthogonal functions. The second independent solution appears as numerator and the orthogonal functions as denominator in an associated continued fraction that is shown to converge to the Riesz–Herglotz–Nevanlinna transform of the measure; the approximants interpolate this function in the Hermite sense and the algorithm is directly related to the Nevanlinna–Pick algorithm that generalizes the Schur algorithm for the polynomial case.

It is not possible to give in this review a complete account of all the ground covered: there is a wealth of excellent written material concerning quite a number of aspects (keywords are: reproducing kernels, Laurent–Padé approximants, (weak) convergence, ratio and root asymptotics, Favard type theorem, etc.).

The chapter on applications gives a good impression of the broad applicability of the methods: linear prediction, Pisarenko modeling, lossless inverse scattering, network synthesis, H_∞ problems (covering H_∞ control, Hankel operators and Hankel norm approximation).

To summarize: an excellent book which is indispensable for anyone who wants to loosen the shackles of the fixed pole situation present in the ordinary theory on the real line and the unit circle. To top that, the price makes it possible for everyone working in the field to buy a personal copy.

References

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